Thermodynamic Properties and Hall Currents in MPD Accelerators

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Theme

THIS paper presents a study of the relationship between local thermodynamic properties, magnetic field strengths, and Hall currents in the arc region of magnetoplasmadynamic (MPD) accelerators. Current density component equations are developed from a generalized Ohm's law which includes Hall current and ion slip terms. Assumptions are made which allow the current density equations to be simplified. A general, nonequilibrium ionization equation is used with these simplified equations to define the local Hall current density in terms of the applied current, the applied magnetic field strength, and the local thermodynamic properties. Spectral intensities are measured and used to calculate electron temperature and density profiles in an experimental accelerator. Total Hall current circulating in the arc region is measured and compared with calculated values.

Contents

Analysis: For the case in which pressure and temperature gradients may be neglected, the generalized Ohm's law may be written as,[1]

$$E' = \frac{1}{\sigma} J + \chi (J \times B) - \psi (J \times B) \times B \qquad (1)$$

where E' is defined as $E + v \times B$, and J and B are the current density and magnetic field strength vectors, respectively. The plasma coefficients σ, χ, and ψ are defined following. This equation and its application to the arc region of an MPD accelerator may be simplified considerably. We assume the accelerator has concentric circular electrodes, therefore the azimuthal electric field is zero, that is $E_\theta = 0$. Secondly, we assume the accelerator cross-section does not change rapidly in the arc region, and that terms involving the radial velocity component may be neglected, that is $v_r \simeq 0$. We also assume that induced magnetic fields are small, and that products of the azimuthal magnetic field strength are negligible, that is $B_\theta \simeq 0$. These assumptions are applicable to a large class of accelerators. Finally, we assume the external magnetic field is applied only in the axial direction, and that terms involving the radial magnetic field strength may be neglected, that is $B_r \simeq 0$. This assumption is valid for the experiments described in this paper.

If these four assumptions are made, the azimuthal current density component equation becomes

$$J_\theta = \frac{\chi \sigma B_z J_r}{1 + \psi \sigma B_z^2} \qquad (2)$$

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It has been shown previously that a relatively simple form of the plasma coefficients may be used to evaluate the current density equations for electron temperatures and densities typical of the present experiments.[1] Therefore, the scalar conductivity is defined as $\sigma = e^2 n_e / m_e (\epsilon_{ea}^{-1} + \epsilon_{ei}^{-1})$, the Hall coefficient may be written as $\chi = 1/en_e$, and the ion slip coefficient is given by $\psi = 2(1-\alpha)^2 \epsilon_{ia} / n_e m_a$. The ionization fraction is defined as $\alpha = n_e / (n_i + n_a)$. ϵ_{ea}, ϵ_{ei}, and ϵ_{ia} are the mean times between electron-atom, electron-ion and ion-atom collisions, respectively. The collision times may be shown to be functions of the local thermodynamic properties only. Therefore the local values of the Hall current density may be determined from Eq. (2) if the radial current density profile, the applied magnetic field strength, and the local thermodynamic properties T_e, n_e, T_i, and α are known.

For our experimental device the discharge has a concentrated attachment at the cathode and is diffuse at the anode. We assume that the discharge has a tight central core in which the current is purely axial. Outside the core the applied current is assumed to be purely radial, and axially uniform, that is $J_r = I/(2\pi r L)$, where I is the total applied current, and L is the arc region length.

The plasma of interest in this study is in a density and temperature range where neither the Saha equation for local thermal equilibrium nor the coronal approximation is valid.[2,3] Bohn has developed a more general non-equilibrium thermodynamic relationship based on an assumption of collisional ionization and collisional and radiative recombination.[4] This equation approaches the Saha equation for high electron density and the coronal equation for very low electron density. For a steady-state plasma, the number densities of two consecutive ionization stages may be related by the following ionization equation

$$\zeta_j = \frac{n_{j+1}}{n_j} = e^{-xj} \Big[6.531 \times 10^{-15} \frac{U_j}{U_{j+1}} \frac{n_e}{T_e 3/2} + \epsilon_j^{3/2} z_{j+1}$$
$$\times (0.429 + 05 \ln x_j + \frac{0.469}{x_j^{1/2}}) \Big]^{-1} \qquad (3)$$

where n_j is the number density of the jth ionization stage, $x_j = 1.16 \times 10^4 (\epsilon_j / T_e)$, ϵ_j is the ionization potential of the jth stage in eV, and Te is the electron temperature in °K. The electron number density, n_e is in m^{-3}, U_j is the internal partition function of the jth species, and z_{j+1} is the charge number of the $j+1$ species. We may combine the definition for ζ_j with an expression for the total pressure as the sum of partial pressures and a statement of quasi-neutrality to obtain an equation for electron density as a function of electron and ion temperatures.[4] For a plasma containing few multiply-charged ions this expression may be reduced to

$$n_e = \frac{P}{KT_i} \left[\frac{\zeta_1 + 2\zeta_1\zeta_2}{1 + \zeta_1 (1 + T_e/T_i)} \right] \qquad (4)$$

The ratio of the azimuthal Hall current density to the applied radial current density given by Eq. (2) may be shown to have a maximum value as B_z varies. A comparison of this

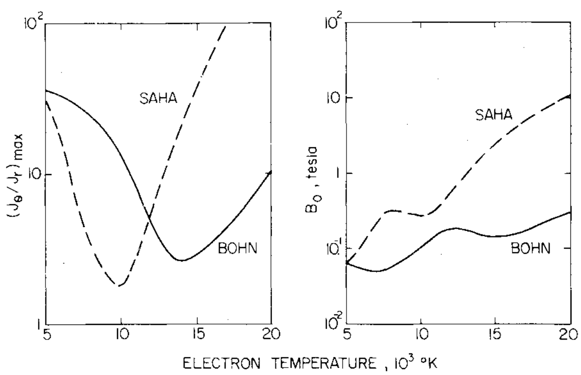

Fig. 1 Comparison of $(J_\theta/J_r)_{max}$ and B_0 for Saha and Bohn equations, $P = 10^{-3}$ atm, $T_e/T_i = 10$.

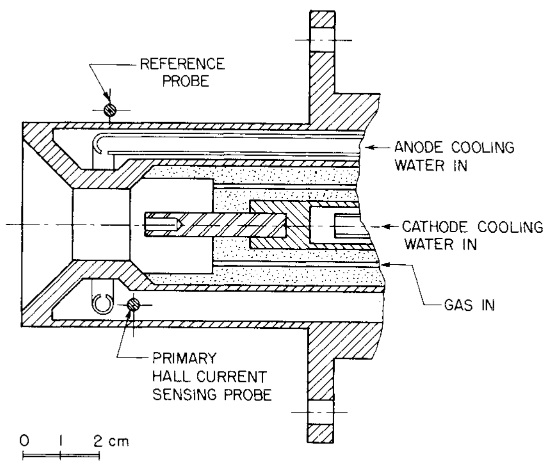

Fig. 2 Section view of accelerator.

maximum current density ratio and the critical magnetic field strength for the Saha and Bohn equations is given in Fig. 1. This figure demonstrates that values based on the thermal equilibrium assumption are significantly different from those based on the more general ionization equation.

Experiments: Radial profiles of electron temperature and density were inferred from spectral intensity measurements in the arc region of a typical plasma accelerator. Total Hall current was measured as a function of externally applied magnetic field strength. Argon was the test gas, with the flow rate varied from 0.04 to 0.12 g/sec. Background tank pressures varied from 3.5 to 7.5×10^{-2} Torr. Input current varied from 70 to 100 A. The applied magnetic field strength ranged from 0.0 to 0.24 T. A section view of the accelerator is shown in Fig. 2.

Magnetic fields induced by the circulating Hall current were measured with semiconductor probes located near the arc region. A discussion of the optical system and the technique used to measure the radial profiles of spectral intensity is given in Ref. 5.

Electron temperatures in the arc region were calculated from the measured spectral profiles using a technique con-

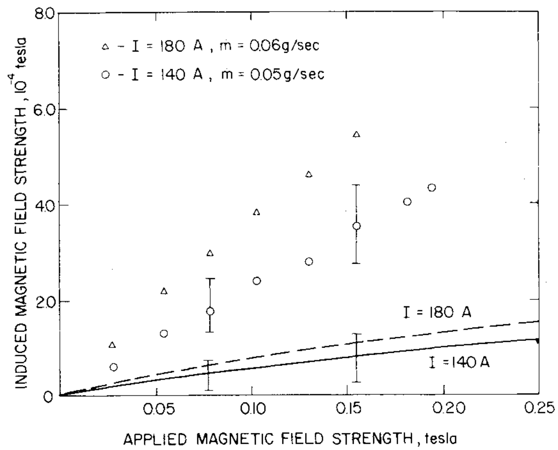

Fig. 3 Comparison of measured and predicted induced field strength as functions of applied magnetic field strength.

sistent with the previously mentioned collisional-radiative model.[2] For fixed operating conditions, the electron temperature was found to be uniform over the arc region. For the range of external operating parameters considered, the measured electron temperature varied from 16.3×10^3 °K to 19.5×10^3 °K. For collisional-radiative recombination the electron density may be determined from absolute intensity measurements if the electron temperature is known.[3] For a typical test run ($\dot{m} = 0.04$ g/sec, $I = 120$A, $B = 0.026$ T) a good fit for the electron density profile was given by $n_e = 1.90 \times 10^{21} \exp(-4.5r^2/r_a{}^2)$, where r_a is the anode radius.

Typical measured values of induced magnetic field strength are compared with theoretically derived values in Fig. 3. The theoretical fields induced at the probe location were determined by calculating Hall current density profiles based on the measured temperature and density profiles. In general, measurements of the induced field strength were larger than the predicted values by a factor ranging from two to five. Both measured and predicted values increased with applied field strength, and both showed a decreasing rate of change with increase in applied field strength. Neither predicted nor measured values passed through a maximum over the range of applied field strengths tested.

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